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# Extraction of Long-term rhythmic structures using the Empirical Mode Decomposition

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## ABSTRACT

Long-term musical structures provide information concerning rhythm, melody and the composition. Although highly musically relevant, these structures are difficult to determine using standard signal processing techniques. In this paper, a new technique based on the time-domain empirical mode decomposition is explained. It decomposes a given signal into its constituent oscillations that can be modified to produce a new version of the signal. It enables us to analyse the long-term metrical structures in musical signals and provides insight into perceived rhythms and their relationship to the signal. The technique is explained, and results are reported and discussed.

**Keywords**: Empirical Mode Decomposition, Music Analysis, Long-term Structures, Rhythm, Tempo tracking.

## 1. INTRODUCTION

Extraction of musically relevant structures is an essential task prior to musical content analysis. Analysis of the individual melodies, themes, phrases and notes provide a better perspective of the signal. Different frequency bands carry different levels of information. So it is potentially useful to separate the high frequency noise and transients from the middle frequency harmonics and melodic information and low frequency long-term information. We can then process each part separately. We can also modify or change the content of each part and recombine them to produce a modified version of a given signal.

This paper concerns applying a new technique based on the time-domain Empirical Mode Decomposition (EMD) to determine a hierarchical structure of the signal. The signal is decomposed into a summation of zero-mean AM-FM[1] (Amplitude or Frequency Modulated) components, called the Intrinsic Mode Functions (IMF) [1].

The Fourier transform has two severe restrictions: stationarity and linearity. The wavelet transform, which is a multiple-resolution STFT, can be used to analyse the non-stationary signals, but still assumes the linearity

[1] The Modes may contain Amplitude or Frequency Modulated components.

condition. Alternatively, EMD can be used as a reliable means to analyse non-linear and non-stationary signals.

Lerdahl and Jackendoff [2] define four main musical structures:

- Grouping structure to explain the segmentation of music as motives, phrases, themes, etc…
- Metrical structure, the structure of the strong and the weak beats.
- Time-span reduction, which is the rhythmic structure according to which the fundamental frequencies are heard.
- Prolongational reduction which expresses the sense of tension and relaxation in music and shows the harmonic and melodic continuity and progression.

Here we show that using the EMD, hierarchic rhythmic structures can be extracted, where each empirical mode is a reduced version of the preceding modes. EMD can be used to obtain both short-term features like fundamental frequency, chord and onset, and long-term structures like rhythm and tempo contours [3]. One advantage of directly finding the long-term structures, rather than calculating them through temporal analysis (e.g. determining tempo through the onsets) is to avoid the errors in temporal measurements transfer to the long-term estimations.

Other audio signal processing applications of the empirical modes may be segregation of polyphonic texture, filtering [4], noise reduction [5] and compression of the audio signal by omission of the perceptually unimportant modes.

This paper is organized as follows. Section 2 introduces the EMD and explains the algorithm. Simulated experiments on various audio signals are described in Section 3. We demonstrate that these experiments reveal the long-term structures as described by Lerdahl and Jackendoff [2]. Section 4 concludes the article with a discussion of future research.

## 2. EMPIRICAL MODE DECOMPOSITION

Empirical Mode Decomposition is an adaptive tool to analyse non-linear or non-stationary signals which segregates the constituent parts of a signal based on its local behaviour. No pre-processing is required since it is able to analyse non-zero mean signals, and is suitable to analyse the riding waves which may have no zero-crossing between two consecutive extrema. It can be used as a filter bank [4], and for signal period analysis [6].

Unlike the Fourier and wavelet transforms, EMD has no fixed basis. It is similar to PCA and ICA in that the basis for the decomposition is signal-dependent. EMD involves calculating the IMFs for the signal, where the IMFs must satisfy the following two conditions:

1) The number of extrema and the number of zero-crossings must either be equal or differ at most by one. That is, there is only one extremum between two zero-crossings.

2) At any point, the mean value of each IMF must be zero.

The Intrinsic Mode Functions are calculated by performing the following sifting process [1]:

1- Through local analysis of the signal, all the minima and maxima are located. An interpolation function connects all the maxima; the same is done for the minima. This gives the upper and lower envelopes.
2- The local mean (mean of the upper and lower envelopes) is calculated: $m_1$
3- The local mean is subtracted from the original signal to obtain the local details:

$$h_1 = X(t) - m_1 \tag{1}$$

4- $h_1$ then becomes the new signal and the sifting process, steps 1 through 3, are repeated until the mean of the local detail becomes negligible, due to a stopping criterion. A threshold must be assigned for this variance between the two consecutive results:

$$Var = \sum_{t=0}^{T} \left[ \frac{\left| \left( h_{1(k-1)}(t) - h_{1k}(t) \right) \right|^2}{h^2_{1(k-1)}(t)} \right] \tag{2}$$

Where, $h_{1k}(t)$ is the result of the $k^{\text{th}}$ iteration on equation (1) and $T$ is the measurement period. The threshold is normally set between 0.05 and 0.3 [1, 7].
The maximum number of iterations is another stopping criterion. Its value can be chosen between 4 and 10 to yield meaningful modes [7]. A high value for the maximum number of iterations causes extra calculations and may lead to over-decomposition of the signal.

Once a stopping criterion is met, the first residue $r_1$ is obtained. It is the first IMF.

5- The residue in step 4 is subtracted from the signal for the first residue and from the previous mode for the others. Then steps 1-5 are performed to calculate the next IMF.
6- The algorithm iterates on step 5, until it becomes a monotonous function that cannot produce any new IMF.

It has been shown that, for estimation of the signal envelopes, using cubic spline interpolation yields better results than linear or polynomial interpolations [7]. The resulting curve is sufficient for estimation of the local mean, while avoiding the 'over-decomposition'.

The original signal may be re-constructed using the following summation:

$$\sum_{i=1}^{n} IMF(i) + r_n \qquad (3)$$

Where $IMF(i)$ is the $i^{th}$ Intrinsic Mode Function; $n$ is the number of the Modes; and $r_n$ is the last residue (residue of the $n^{th}$ mode).

In practice the interpolation in step 1 will not be perfect. This is due to insufficient data, and the uncertainty in the end-values of the envelopes. Furthermore, it is important to have enough samples for the peak detection step. Otherwise we will face the resulting error in the calculated modes. The influence of sampling on the behaviour of EMD is elaborated in [8].

There are 3 main issues with this procedure: how to define the stopping criteria, how to detect peaks, and how to deal with end effects in construction of the envelope.

The end effect has been discussed in several previous papers on the EMD [1, 4-6]. It pertains to the difficulty in estimation of the bottom and top envelopes of a signal near the beginning or end of the signal. The envelopes are typically created using cubic spline interpolation, but at the endpoints there is not enough data to perform a cubic spline.

Huang [1] suggested adding false peaks such as to yield typical waveforms at each end, with envelopes starting from zero to the first peak and from the last peak to zero. If the peaks occur at $t(P_1), t(P_2)...$, then this may be accomplished by setting a peak at:

$$t(P_0) = t(P_1) - [t(P_2) - t(P_1)] \qquad (4)$$

And similarly, setting a peak after the last peak. It may be necessary to add several peaks near each endpoint. Other methods include setting a peak at the first data point with amplitude equal to that of the first data point, this guarantees that the envelope converges onto or near the data. We have tried both methods and several more, but none guarantees success.

The accuracy of the peak detection algorithm also significantly affects results. Peaks can be missed, false peaks can be added, and peak amplitudes can be miscalculated. These result in a poor envelope. A single false peak or grossly miscalculated peak amplitude can result in an error in the envelope which perpetuates, and may even grow, through subsequent shiftings and calculation of modes.

Detection of peaks is improved by having a high sample rate. A sample rate of Fs is sufficient to resolve frequencies up to Fs/2, but that implies that frequency content near Fs/2 will have only 2 points per period. This makes accurate detection of peaks very difficult. One possible solution is preprocessing, i.e., perform an FFT, remove all the high frequency content, and then perform an inverse FFT. This may smooth out the most difficult peaks.

The stopping criteria for sifting is less significant, in that different choices of stopping criteria will yield different results, but not necessarily incorrect results. The main criteria defined by Huang are that the component has no riding waves and that the mean envelope is zero [1]. No riding waves simply mean that there are no maxima below zero and no minima above zero. This also implies that the number of zero crossings differ from the total number of maxima and minima by at most one. The second criterion for stopping the sifting, that the mean envelope is zero, is far more difficult. Errors in peak detection and end effects may result in significant deviation of the mean envelope, and hence lead to more sifting.

The implementation of the EMD that has been performed here is based on freely available MATLAB code by Rilling, et. al. [4, 7]. Spline interpolation has been used with false peaks added near the endpoints. The stopping criterion in Equation (2) was typically set to 0.1, and no pre-processing was applied.

## 3. EXPERIMENTS & RESULTS

Using a computer with a sound card, and an ordinary microphone, samples of 16-bit precision at a sampling rate of 44.1 kHz were acquired. The samples were performed by the first author on a Persian Santur instrument. The Santur is a trapezoidal string instrument, played by a pair of delicate hammer sticks. It is often referred to as a Hammered Dulcimer in English [9].

### 3.1 Experiment 1

Figure 1 shows the scores for an array of the following notes: A3-C4-E4-A4-C5-E5-A5. The fundamental frequencies are 220, 261.6, 329.6, 440, 523.25, 659.25 and 880 Hertz respectively [10].

Figure 2 shows the spectrum of the two-octave A minor arpeggio played on a Santur. A 256 point window has been used. The change in the harmonic content at the onset of each note can be clearly seen.

The same signal has been analysed by the EMD. With a Maximum Iterations of 20, the arpeggio is decomposed to 13 empirical modes, marked as F1-F13 and a residue (figure 3).

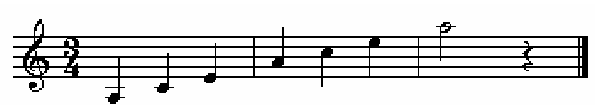


Figure 1 A two-octave A minor arpeggio

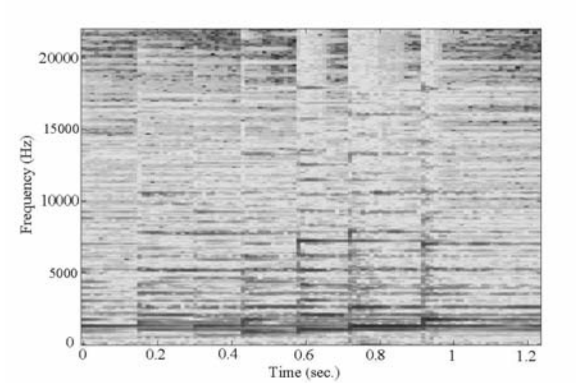



Figure 2 Spectrum of a two-octave A minor arpeggio.

The EMD acts as an adaptive filter bank. The first few IMFs contain the high frequency noise and the harmonic information, and the lower modes show the long-term behaviour of the signal. Although, here only the first 7 IMFs can be heard, the next modes still convey important information. They tell us about the metrical and rhythmic structures. For example, comparing the IMF 12 with the signal, we observe that the peaks of the sinusoid in IMF 12 occur close to A onsets. So IMF12 peaks can be interpreted as the strong beat, i.e. the metrical structure; and IMF 13 separates the two arpeggios into measures, each happening in a quarter-cycle of the oscillation in IMF13. It shows the rhythmic structure as described by Lerdahl and Jackendoff [2].

The signal can be reconstructed, with summing up all the modes and the residue according to equation 3.

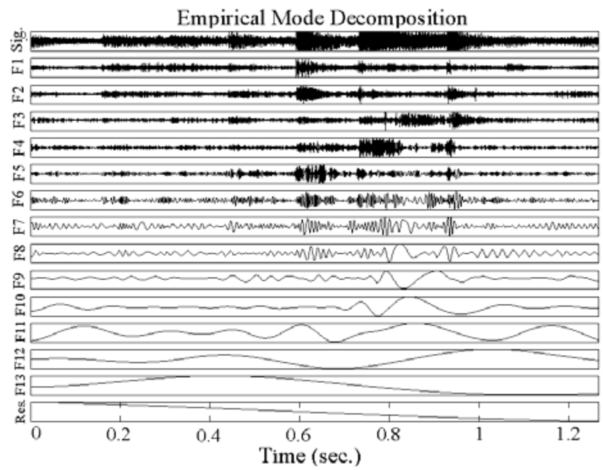



Figure 3 Decomposition of the sample in figure 1: signal, its 13 IMFs and the residue

### 3.2 Experiment 2

In the second test, two A4-C5 and C5-E5 notes were played several times as a retarding rhythmic pattern (figure 4), where the tempo is gradually decreasing. Tempo is the speed of the rhythm of a piece, measured as the number of beats per minute. A tempo tracking system is explained in [11].

Figure 5 shows the EMD results with a Maximum Iterations of 5. Inspection of the IMFs in this figure allows one to speculate on the relationship between the IMFs and the harmonic content of the original signal. By comparing the frequency content of the IMFs with the frequencies of the note sequence, it can be seen that IMF1 has strong frequencies that match the 5th harmonic of A4 and the 4th harmonic of C5; similarly, IMF2 has the 5th harmonic of C5; IMF3, 2nd harmonic of A4; IMF4, F0 of C5; IMF5, F0/2 of C5; IMF6 F0/2 of A4.

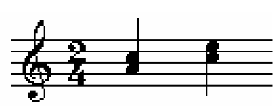


Figure 4 A retarding sequence of A4-C5 and C5-E5 chords

The half-pitch components in the signal could be interpreted as the sympathetic vibration of A3 strings. The sympathetic vibration happens when a string is not played, but vibrated by another sound of the same F0 or a multiple of that. Further research is required to verify these conjectured relationships.

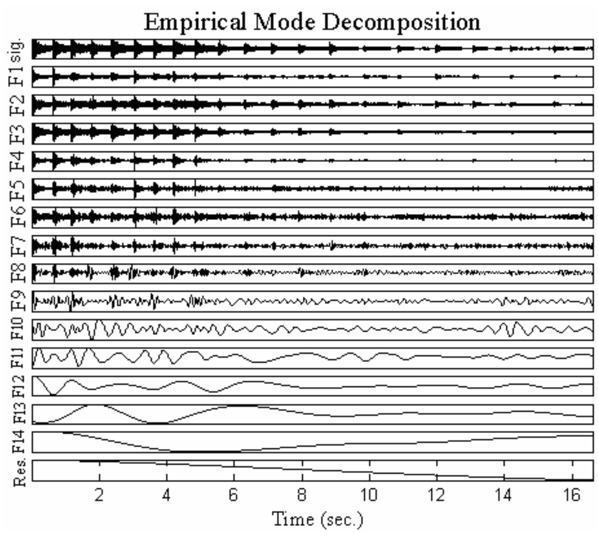
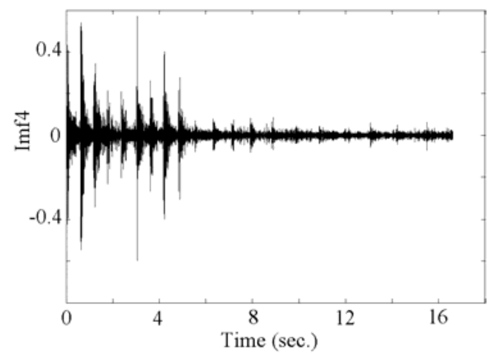



Figure 5 A decreasing tempo sequence of A4-C5 and C5-E5 notes: signal, its 14 IMFs and the residue

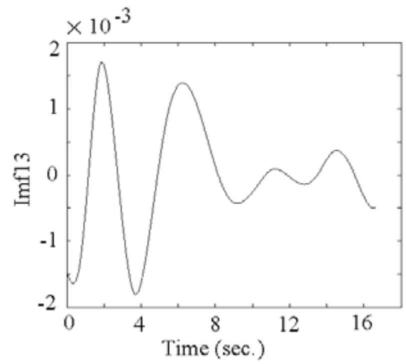
The explanation for this phenomenon is given with the fact that the EMD acts as an adaptive filter bank. With increasing the mode index, frequencies of the oscillations become lower. A higher value for the maximum number of iterations would decompose the signal into more modes, but would significantly increase the amount of processing required. Too high of a maximum value may also lead to over-decomposition of the signal.

The period of IMF11, which is changing through time, shows the onset times. And with decreasing the tempo, the periods of IMF13 & IMF14 increase. So they might be used for tempo tracking. IMF13 has a period 6 times the distance of the first 2 notes, though it is arranging the notes in groups of 6 similar to the time span segmentation suggested by Lerdahl and Jackendoff [2]. The same can be said for IMF14 but with a larger period (10 notes). The residue shows a decreasing trend as the tempo decreases. Figures 6-a through 6-c show IMF4 (C5's F0), IMF13 and IMF14 in a larger view.

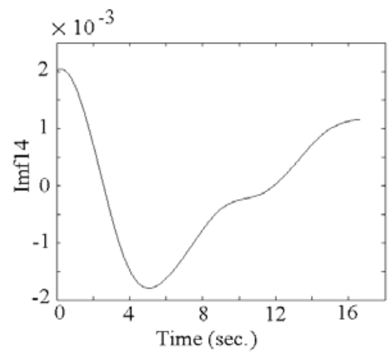
Similarly, other tests were done on increasing and decreasing tempo patterns of an A4 note and on some melodic patterns. They reinforce this statement that the last few modes in an EMD decomposition of a signal follow the rhythmic and metrical structure.




(6-a)




(6-b)




(6-c)

Figure 6 IMF4, IMF13 and IMF14 of figure 3 a) IMF4 b) IMF13 c) IMF14

Using the EMD, a rhythmic analysis of the signal can be performed. The obtained modes are hierarchically ordered and the EMD operates as a filter bank with noise and higher frequency components in the first few IMFs, and lower frequency components in the lower modes.

## 4. CONCLUSIONS

This work is concerned with applying the Empirical Mode Decomposition to extract meaningful musical structures from audio. The EMD is a powerful means for the analysis of nonlinear non-stationary signals. It decomposes the signal to a summation of zero-mean AM-FM components, called Intrinsic Mode Functions. EMD has no analytical representation and is based on the local behaviour of the signal. It can be used for the analysis of long-term structures which are difficult to determine using standard frequency domain or wavelet techniques.

Using the EMD, the signal is decomposed into a set of hierarchically ordered modes, where each empirical mode is a reduced version of the preceding modes (figures 3 and 4). EMD operates as a filter bank with noise and higher frequency components in the first few IMFs, and lower frequency components in the lower modes. This hierarchical representation of a musical piece can be used for noise reduction, or adaptive segregation of different frequency bands in an audio signal. Future work may be on automatic analysis of the long-term structures like the Scale and the rhythm in a musical piece. This will assist in automated music labeling. Also, the IMFs can be individually modified or changed, to produce a modified version of the signal.

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